

On the very high energy spectrum of the Crab pulsar.

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ABSTRACT

In the present paper we construct a self-consistent theory, interpreting the observations of the MAGIC Cherenkov Telescope of the very high energy (VHE) pulsed emission from the Crab pulsar. In particular, on the basis of the Vlasov's kinetic equation we study the process of the quasi-linear diffusion (QLD) developed by means of the cyclotron instability. This mechanism provides simultaneous generation of low (radio) and VHE (0.01-25 GeV) emission on the light cylinder scales, in one location of the pulsar magnetosphere. A different approach of the synchrotron theory is considered, giving the spectral index of VHE emission ($\beta = 2$) and the exponential cutoff energy (23 GeV) in a good agreement with the observational data.

Subject headings: instabilities - plasmas - pulsars: individual (PSR B0531+21) - radiation mechanisms: non-thermal

1. Introduction

The recent observations of the MAGIC Cherenkov telescope (Aliu et al. 2008), reveal several characteristic features of VHE emission of the Crab pulsar. In particular, pulsed γ -rays above 25 GeV is detected showing a relatively high energy cut-off, which indicates that emission happens far out in the magnetosphere (Aliu et al. 2008). In general, for explaining γ -ray production in the pulsar magnetosphere two different models are applied: the so-called 'polar cap' and 'outer gap' models. In the former it is assumed that the VHE emission is generated at the polar cap (e. g. Daugherty & Harding (1982)), but it can not coincide in phase with the entire radio emission. And in the outer gap models the generation of the VHE radiation happens in the outer gap region (e. g. Romani & Yadigaroglu (1995)), but to our knowledge there is no mechanism of generation of radio emission. The pulsar emission model that underlies our work principally differs from polar cap and outer gap models. To explain the origin of the very high energies observed by the MAGIC Cherenkov Telescope (Aliu et al. 2008), we rely on the pulsar emission model first developed by

Machabeli & Usov (1979) and Lominadze et al. (1979). According to these works, in the electron-positron plasma of a pulsar magnetosphere the low frequency cyclotron modes, on the quasi-linear evolution stage create conditions for generation of the high energy synchrotron emission. A special interest deserves the coincidence of signals from different frequency bands ranging from radio to X-rays (Manchester & Taylor 1980). Investigations of last decade have shown that the aforementioned coincidence takes places in the very high energy domain (0.01 MeV-25 GeV) as well (Aliu et al. 2008). In the framework of the present paper, generation of low and high frequency waves is a simultaneous process and it takes place in one location of the magnetosphere, which explains the observed pulse phase coincidence of the low and VHE signals. Consequently, we suppose that generation of phase-aligned signals from different frequency bands is a simultaneous process and takes place in one location of the pulsar magnetosphere. This consideration, automatically excludes the inverse Compton scattering and the curvature radiation mechanisms respectively, which are not localized (Machabeli & Osmanov 2009, 2010). It is worth noting that coincidence of pulse phases might be achieved by means of caustic effects (Morini 1983; Romani & Yadigaroglu 1995; Dyks et al. 2004).

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It is well known that close to the pulsar surface due to very strong magnetic fields, magnetospheric particles emit efficiently and the corresponding cooling timescale is short compared to the typical kinematic timescales of particles. Therefore, transversal energy loss becomes extremely efficient, consequently electrons and protons lose their perpendicular momenta and very rapidly transit to their ground Landau states and the distribution function becomes one dimensional. This means that one needs a certain mechanism, leading to the creation of the pitch angles restoring the synchrotron radiation. The main mechanism of wave generation in plasmas of the pulsar magnetosphere is the cyclotron instability (Kazbegi et al. 1992). During the quasi-linear stage of the instability, a diffusion of particles arises along and across the magnetic field lines. Therefore, the resonant electrons acquire transverse momenta and, as a result start to radiate in the synchrotron regime.

Recently we applied the method of the quasi-linear diffusion to the Crab pulsar to explain the VHE emission observed by the MAGIC Cherenkov Telescope (Aliu et al. 2008). In (Machabeli & Osmanov 2009) we found that on the light cylinder (a hypothetical zone, where the linear velocity of rigid rotation exactly equals the speed of light) length-scales the cyclotron modes are generated, provoking the re-creation of the pitch angles and the subsequent synchrotron radiation in the VHE ($> 25\text{GeV}$) domain. The quasi-linear diffusion guarantees the observationally evident fact of coincidence of signals in the low and the VHE bands (Machabeli & Osmanov 2009), meaning the simultaneous generation of these radiation domains in one location of the pulsar magnetosphere. As it has been shown, neither the curvature radiation nor the inverse Compton scattering may provide the above mentioned coincidence. This particular problem was considered in (Machabeli & Osmanov 2010), where analyzing the inverse Compton scattering, it has been demonstrated that for reasonable physical parameters even very energetic electrons are unable to produce the photon energies of the order of 25GeV . Studying the curvature radiation, we have found that the curvature drift instability (see Osmanov et al. 2008; Osmanov et al. 2009) efficiently rectifies the magnetic field lines making

the role of the curvature emission process negligible (Machabeli & Osmanov 2010).

According to the theory of the synchrotron emission (Bekefi & Barrett 1977; Ginzburg 1981) the spectral index is usually less than 1 (in most of the cases it equals $1/2$), although it is observationally evident that for VHE pulsed radiation from the Crab pulsar the corresponding value is of the order of 2 (Aliu et al. 2008). In the standard theory of the synchrotron radiation, unlike the present model, it is assumed that along the line of sight the magnetic field is chaotic, leading to the broad interval (from 0 to π) of the pitch angles. Contrary to this scenario, as we have already outlined, in the pulsar magnetospheres the magnetic field is very strong and pitch angles rapidly vanish. The present model provides all necessary conditions for re-creation of the pitch angles, consequently restricting their values. In the framework of the paper we study the spectral index, which, for reasonable magnetospheric parameters is of the order of 2 for the high energy domain.

The paper is organized as follows. In Section 2 we consider the emission model, in Sect. 3 we study the synchrotron radiation spectrum, in Sect. 4 we discuss our results and in Sect. 5 we summarize them.

2. Emission model

According to the works of Sturrock (1971) and Tadamaru (1973) due to the cascade processes of the pair creation a pulsar's magnetosphere is filled by electron-positron plasma with an anisotropic one-dimensional distribution function (see Fig. 1 from Arons (1981)) and consists of the following components: the bulk of plasma with an average Lorentz-factor $\gamma \sim \gamma_p$, a tail - γ_t , and the primary beam with $\gamma \sim \gamma_b$. While considering the eigenmodes of electron-positron plasma having small inclination angles with respect to the magnetic field, one has three branches, two of which are mixed longitudinal-transversal waves ($lt_{1,2}$). The high frequency branch on the diagram $\omega(k_{\parallel})$ begins with the Langmuir frequency and for longitudinal waves ($k_{\perp} = 0$), lt_1 reduces to the pure longitudinal Langmuir mode. The low frequency branch, lt_2 , is similar to the Alfvén wave. The third t mode, is the pure transversal wave, the electric component of which \mathbf{E}^t is perpendicular to the

plane of the wave vector, and the magnetic field, $(\mathbf{k}, \mathbf{B}_0)$. The vector of the electric field, \mathbf{E}^{lt_1, lt_2} is located in the plane $(\mathbf{k}, \mathbf{B}_0)$. When $k_\perp = 0$, the t -mode merges with the lt waves and the corresponding spectra is given by (Kazbegi et al. 1992)

$$\omega_t \approx kc(1 - \delta), \quad \delta = \frac{\omega_p^2}{4\omega_B^2 \gamma_p^3} \quad (1)$$

where k is the modulus of the wave vector, c is the speed of light, $\omega_p \equiv \sqrt{4\pi n_p e^2/m}$ is the plasma frequency, e and m are the electron's charge and the rest mass, respectively, n_p is the plasma density, $\omega_B \equiv eB/mc$ is the cyclotron frequency and B is the magnetic field induction.

The distribution function is one-dimensional and anisotropic and plasma becomes unstable, which can lead to excitation of the aforementioned waves. The beam particles undergo drifting perpendicularly to the magnetic field due to the curvature, ρ , of the field lines. The corresponding drift velocity is given by

$$u_x \equiv \frac{cV_\parallel \gamma_{res}}{\rho\omega_B} \quad (2)$$

where V_\parallel is the component of velocity along the magnetic field lines and γ_{res} is the Lorentz factor of the resonant particles. Both of these factors (the one-dimensionality of the distribution function and the drift of particles) might cause generation of eigen modes in the electron-positron plasma if the following resonance condition is satisfied (Kazbegi et al. 1992)

$$\omega - k_\parallel V_\parallel - k_x u_x + \frac{s\omega_B}{\gamma_{res}} = 0, \quad (3)$$

where k_x is the wave vector's component along the drift and $s = 0, \pm 1, \pm 2, \dots$

For $s = 0$ one has a hollow cone of the modified Cherenkov radiation (Kazbegi et al. 1992; Lyutikov et al. 1999; Shapakidze et al. 2003). For the Crab pulsar one has the core emission, being a result of the anomalous Doppler effect ($s = -1$). Our aim is to interpret the results of the MAGIC Cherenkov Telescope (Aliu et al. 2008) therefore, in the present paper we consider the aforementioned resonance condition $s = -1$.

During the generation of t or lt modes by resonant particles, one also has a simultaneous feedback of these waves on the electrons

(Vedenov et al. 1961). This mechanism is described by QLD, leading to the diffusion of particles as along as across the magnetic field lines. The process of QLD in the external magnetic field is examined in a series of books (Melrose & McPhedran 1991; Akhiezer 1967). Generally speaking, at the pulsar surface relativistic particles efficiently lose their perpendicular momenta via synchrotron emission in very strong ($B \sim 10^{12}$ G) magnetic fields and therefore, they very rapidly transit to their ground Landau state (pitch angles are vanishing). Contrary to this process, QLD leads to creation of the pitch angles by resonant particles and as a result they start to radiate in the synchrotron regime. To explain the observed very high energy emission of the Crab pulsar, it is supposed that the resonant particles are the primary beam electrons with $\gamma_b \sim 10^8$, giving the synchrotron emission in the VHE domain.

When emitting in the synchrotron regime, the resonant particles undergo the radiation reaction force \mathbf{F} , having as longitudinal as transversal components (Landau & Lifshitz 1971):

$$F_\perp = -\alpha_s \frac{p_\perp}{p_\parallel} \left(1 + \frac{p_\perp^2}{m^2 c^2} \right), \quad F_\parallel = -\frac{\alpha_s}{m^2 c^2} p_\perp^2, \quad (4)$$

where $\alpha_s = 2e^2 \omega_B^2 / 3c^2$.

The wave excitation leads to a redistribution process of the particles via QLD. The kinetic equation for the distribution function of the resonant particles can be written as (Machabeli & Usov 1979; Malov & Machabeli 2002):

$$\begin{aligned} & \frac{\partial f^0}{\partial t} + \frac{\partial}{\partial p_\parallel} \{F_\parallel f^0\} + \frac{1}{p_\perp} \frac{\partial}{\partial p_\perp} \{p_\perp F_\perp f^0\} = \\ & = \frac{1}{p_\perp} \frac{\partial}{\partial p_\perp} \left\{ p_\perp \left(D_{\perp, \perp} \frac{\partial}{\partial p_\perp} + D_{\perp, \parallel} \frac{\partial}{\partial p_\parallel} \right) f^0(\mathbf{p}) \right\} + \\ & + \frac{\partial}{\partial p_\parallel} \left\{ \left(D_{\parallel, \perp} \frac{\partial}{\partial p_\perp} + D_{\parallel, \parallel} \frac{\partial}{\partial p_\parallel} \right) f^0(\mathbf{p}) \right\}. \quad (5) \end{aligned}$$

The diffusion coefficients in Eq. (5) are evaluated in the momentum space as (Melrose & McPhedran 1991):

$$\begin{pmatrix} D_{\perp, \perp} \\ D_{\perp, \parallel} = D_{\parallel, \perp} \\ D_{\parallel, \parallel} \end{pmatrix} = \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \frac{\pi^2 e^2 v^2 \sin^2 \psi}{\hbar^2 \omega^2} \frac{|E_k|^2}{4\pi} \times$$

$$\times \delta(\omega(\mathbf{k}) - k_{\parallel} v_{\parallel} + \omega_B/\gamma) \begin{pmatrix} (\Delta p_{\perp})^2 \\ (\Delta p_{\perp})(\Delta p_{\parallel}) \\ (\Delta p_{\parallel})^2 \end{pmatrix}. \quad (6)$$

Where $|E_k|^2/4\pi$ is the density of electric energy in the excited waves and

$$\Delta p_{\perp} = -\frac{\hbar\omega_B}{\gamma v \sin \psi}, \quad \Delta p_{\parallel} = \hbar k_{\parallel}. \quad (7)$$

The evaluation in our case gives

$$\begin{pmatrix} D_{\perp,\perp} \\ D_{\perp,\parallel} = D_{\parallel,\perp} \\ D_{\parallel,\parallel} \end{pmatrix} = \begin{pmatrix} D\delta|E_k|_{k=k_{res}}^2 \\ -D\psi|E_k|_{k=k_{res}}^2 \\ D\psi^2\frac{1}{\delta}|E_k|_{k=k_{res}}^2 \end{pmatrix}, \quad (8)$$

where $D = e^2/8c$.

The pitch-angle acquired by resonant particles during the process of QLD satisfies $\psi = p_{\perp}/p_{\parallel} \ll 1$. Thus, one can assume $\partial/\partial p_{\perp} \gg \partial/\partial p_{\parallel}$ which reduces Eq. (5) to the following form

$$\begin{aligned} \frac{\partial f^0}{\partial t} + \frac{1}{p_{\perp}} \frac{\partial}{\partial p_{\perp}} (p_{\perp} F_{\perp} f^0) = \\ = \frac{1}{p_{\perp}} \frac{\partial}{\partial p_{\perp}} \left(p_{\perp} D_{\perp,\perp} \frac{\partial f^0}{\partial p_{\perp}} \right). \end{aligned} \quad (9)$$

The transversal diffusion leads to the izotropization of the one-dimensional distribution function whereas the force F_{\perp} works against the diffusion. The dynamical process saturates when these effects balance each other. Considering the quasi-stationary state ($\partial f/\partial t = 0$), one finds

$$f(p_{\perp}) = C \exp \left(\int \frac{F_{\perp}}{D_{\perp,\perp}} dp_{\perp} \right) = C e^{-\left(\frac{p_{\perp}}{p_{\perp 0}} \right)^4}, \quad (10)$$

where

$$p_{\perp 0} \approx \frac{\pi^{1/2}}{B\gamma_p^2} \left(\frac{3m^9 c^{11} \gamma_b^5}{32e^6 P^3} \right)^{1/4}. \quad (11)$$

Taking into account the above relation we can find that the mean value of the pitch-angle, $\psi_0 \approx p_{\perp 0}/p_{\parallel}$, is of the order of 10^{-6} .

3. Synchrotron radiation spectrum

Let us consider the synchrotron emission of the set of electrons. If $p_{\perp} f dp_{\perp} dp_{\parallel} dV d\Omega_{\tau}$ is the number of emitting particles in the elementary dV volume, with momenta from the intervals $[p_{\perp}, p_{\perp} +$

$dp_{\perp}]$ and $[p_{\parallel}, p_{\parallel} + dp_{\parallel}]$, and with the velocities that lie inside the solid angle $d\Omega_{\tau}$ near the direction of $\vec{\tau}$. Then the emission flux of the set of electrons is given by (Ginzburg 1981)

$$F_{\epsilon} = \int I_{\epsilon} p_{\perp} f dp_{\perp} dp_{\parallel} dV d\Omega_{\tau}, \quad (12)$$

where I_{ϵ} is the Stokes parameter, which is additive in this case, as the observed synchrotron radiation wavelength λ is much less than the value of $n^{-1/3}$ - the average distance between particles, where n is the density of plasma component electrons. Taking into account that

$$\int p_{\perp} f^0 dp_{\perp} = f_{\parallel}(p_{\parallel}), \quad (13)$$

the integral (16) is easily reduced to

$$F_{\epsilon} \propto \int f_{\parallel}(p_{\parallel}) B \psi \frac{\epsilon}{\epsilon_m} \left[\int_{\epsilon/\epsilon_m}^{\infty} K_{5/3}(z) dz \right] dp_{\parallel}. \quad (14)$$

Here $\epsilon_m \approx 5 \cdot 10^{-18} B \psi \gamma^2 \text{ GeV}$ is the photon energy of the maximum of synchrotron spectrum of a single electron and $K_{5/3}(z)$ is a Macdonald function. After substituting the mean value of the pitch-angle in the above expression for ϵ_m , we get

$$\epsilon_m \simeq 5 \cdot 10^{-18} \frac{\pi^{1/2}}{\gamma_p^2} \left(\frac{3m^5 c^7 \gamma_b^9}{4e^6 P^3} \right)^{1/4} \quad (15)$$

Accordingly, the beam electrons should have $\gamma_b \simeq 6 \cdot 10^8$ to radiate the photons with $\sim 10 \text{ GeV}$ energy. This in turn implies that the gap models providing the Lorentz factors $\sim 10^7$, are not enough to explain the detected pulsed emission. On the other hand, Aliu et al. (2008) confirmed that their observations indicate that emission happens far out in the magnetosphere. One of the real scenarios could be the centrifugal acceleration of electrons, which take place in co-rotating magnetospheres (Machabeli & Rogava 1994; Rogava et al. 2003; Osmanov et al. 2007). Another alternative mechanism of acceleration could be a collapse (e.g. Artsimovich & Sagdeev 1979; Zakharov 1972) of the centrifugally excited unstable Langmuir waves (Machabeli et al. 2005) in the pulsar's magnetosphere.

According to our emission model, the observed radiation comes from a region where the magnetic field lines are practically straight and parallel to

each other, therefore, electrons with $\psi \approx \psi_0$ efficiently emit in the observer's direction.

To find the synchrotron flux in our case, we need to know the one-dimensional distribution function of the emitting particles f_{\parallel} . Let us multiply both sides of Eq. (5) on p_{\perp} and integrate it over p_{\perp} . Taking into account that the distribution function vanishes at the boundaries of integration, Eq. (5) reduces to

$$\frac{\partial f_{\parallel}}{\partial t} = \frac{\partial}{\partial p_{\parallel}} \left(\frac{\alpha_s}{m^2 c^2 \pi^{1/2}} p_{\perp 0}^2 f_{\parallel} \right). \quad (16)$$

Considering the quasi-stationary case we find

$$f_{\parallel} \propto \frac{1}{p_{\parallel}^{1/2} |E_k|}. \quad (17)$$

For $\gamma\psi \ll 10^{10}$, a magnetic field inhomogeneity does not affect the process of wave excitation. The equation that describes the cyclotron noise level, in this case, has the form (Lominadze et al. 1983)

$$\frac{\partial |E_k|^2}{\partial t} = 2\Gamma_c |E_k|^2 f_{\parallel}, \quad (18)$$

where

$$\Gamma_c = \frac{\pi^2 e^2}{k_{\parallel}} f_{\parallel}(p_{res}), \quad (19)$$

is the growth rate of the instability. Here k_{\parallel} can be found from the resonance condition (3)

$$k_{\parallel res} \approx \frac{\omega_B}{c\delta\gamma_{res}}. \quad (20)$$

Combining Eqs. (16) and (18) one finds

$$\frac{\partial}{\partial t} \left\{ f_{\parallel} - \alpha \frac{\partial}{\partial p_{\parallel}} \left(\frac{|E_k|}{p_{\parallel}^{1/2}} \right) \right\} = 0, \quad (21)$$

$$\alpha = \left(\frac{4}{3} \frac{e^2}{\pi^5 c^5} \frac{\omega_B^6 \gamma_p^3}{\omega_p^2} \right)^{1/4}, \quad (22)$$

which reduces to

$$\left\{ f_{\parallel} - \alpha \frac{\partial}{\partial p_{\parallel}} \left(\frac{|E_k|}{p_{\parallel}^{1/2}} \right) \right\} = const. \quad (23)$$

Taking into account that for the initial moment the major contribution of the lefthand side of the

Eq. (23) comes from $f_{\parallel 0}$, the corresponding expression writes as

$$f_{\parallel} - \alpha \frac{\partial}{\partial p_{\parallel}} \left(\frac{|E_k|}{p_{\parallel}^{1/2}} \right) = f_{\parallel 0}. \quad (24)$$

The distribution function f is proportional to $n \sim 1/r^3$ (here r is the distance from the pulsar), then one should neglect f_{\parallel} in comparison with $f_{\parallel 0}$. Consequently, the above equation reduces to

$$\alpha \frac{\partial}{\partial p_{\parallel}} \left(\frac{|E_k|}{p_{\parallel}^{1/2}} \right) + f_{\parallel 0} = 0. \quad (25)$$

As we can see the function $E_k(p_{\parallel})$ drastically depends on the form of the initial distribution of the primary beam electrons. According to the work, (Goldreich & Julian 1969), a spinning magnetized neutron star generates an electric field which extracts electrons from the star's surface and accelerates them to form a low-density ($n_b = B/Pce$) and energetic primary beam. We only know the scenario of creation of the primary beam, but nothing can be told about its distribution, which drastically depends on the neutron star surface properties and temperature. To our knowledge there is no convincing theory which could predict the form of the distribution function of the beam electrons. Thus, we can only assume that the beam electrons have a power-law distribution

$$f_{\parallel 0} \propto p_{\parallel}^{-n}, \quad (26)$$

and for the energy density of the waves we get

$$|E_k|^2 \propto p_{\parallel}^{3-2n}. \quad (27)$$

The effective value of the pitch angle depends on $|E_k|^2$ as follows

$$\psi_0 = \frac{1}{2\omega_B} \left(\frac{3m^2 c^3}{p_{\parallel}^3} \frac{\omega_p^2}{\gamma_p^3} |E_k|^2 \right)^{1/4}. \quad (28)$$

Using expression (17), (27) and (28), and replacing the integration variable p_{\parallel} by $x = \epsilon/\epsilon_m$, from Eq. (14) we will get

$$F_{\epsilon} \propto \epsilon^{-\frac{2-n}{4-n}} \int x^{\frac{2-n}{4-n}} \left[\int_x^{\infty} K_{5/3}(z) dz \right] dx. \quad (29)$$

According to Aliu et al. (2008) the observed high energy pulsed emission of the crab pulsar is best

described by a power-law spectrum $F(\epsilon) \propto \epsilon^{-2.022}$ in the energy domain (0.01–5) GeV. At $\epsilon = 25$ GeV a measured flux is several times lower, which requires a spectral cutoff somewhere between 5 and 25 GeV.

We assume that the energy of the beam electrons vary between $\gamma_{min} \sim 10^6$ and $\gamma_{max} \sim 10^8$, in which case, we have $(\epsilon/\epsilon_m)_{max} \ll 1$ and $(\epsilon/\epsilon_m)_{min} \gg 1$. Under such conditions the integral (29) can be approximately expressed by the following function

$$F_\epsilon \propto \epsilon^{-\frac{2-n}{4-n}} \exp \left[- \left(\frac{\epsilon}{23} \right)^{1.6} \right]. \quad (30)$$

When $n = 6$ the spectral index, β , of the synchrotron emission equals 2, and the flux $F_\epsilon \propto \epsilon^{-2} \exp[-(\epsilon/23)^{1.6}]$. As we can see our emission scenario predicts the exponential cutoff, with the cutoff energy 23 GeV.

4. Discussion

One of the interesting observational feature of the Crab pulsar is that its multiwavelength emission pulses from low-frequency radio waves up to hard γ -rays ($\epsilon > 25$ GeV) are coincident in phase (Manchester & Taylor 1980; Aliu et al. 2008). Which implies that generation of these waves occurs in the same place of the pulsar magnetosphere. According to the generally accepted point of view, VHE emission is produced either by the Inverse Compton up-scattering or by the curvature radiation. Although it is clear that the aforementioned processes cannot provide the observationally evident coincidence of signals, since they do not have any restriction on the spacial location of emission (area in the pulsar magnetosphere, where the corresponding radiation is produced). This particular problem has been studied by Machabeli & Osmanov (2010). Considering the curvature radiation, we have shown that the curvature drift instability (see Osmanov et al. (2008); Osmanov et al. (2009)) makes the magnetic field lines rectify very efficiently. It has been shown that the increment of the instability is given by [see Eq. 22 in (Osmanov et al. (2009))]

$$\Gamma \approx \left(-\frac{3}{2} \frac{\omega_b^2}{\gamma_{b0}} \frac{k_x u_x}{k_r c} \right)^{1/2} \left| J_0 \left(\frac{k_x u_x}{4\Omega} \right) J_0 \left(\frac{k_r c}{\Omega} \right) \right|, \quad (31)$$

where ω_b is the plasma frequency of the beam component, k_r and k_x are the wave vector's radial component and the component along the rotation axis respectively and J_0 denotes the Bessel function of zeroth order. By considering the perturbation corresponding to $\lambda_x \sim R_{lc}$, $\lambda_r \sim 10^3 R_{lc}$ and the initial curvature of the magnetic field lines being of the order of R_{lc} , where $R_{lc} \sim 10^8$ cm is the light cylinder lengthscale, one can show that the timescale, $t_{CDI} \sim 1/\Gamma$, of the CDI for $\gamma_b \sim 10^8$ equals 1.6s. On the other hand, the instability makes its job (amplifies the toroidal magnetic field) until the excited mode escapes the magnetosphere. This happens in the characteristic timescale $t_{esc} \sim R_{lc}/(v_{ph} \sin \theta)$, where $v_{ph} \equiv \omega/k$ is the phase velocity of the curvature drift wave and $\theta \approx k_r/k_x$ is the inclination of the wave vector with respect to the rotation axis. After taking into account the dispersion relation of the curvature drift mode, $\omega = k_x u_x/2$ (see Osmanov et al. (2008); Osmanov et al. (2009)), it is straightforward to show that $t_{esc} \approx 2.5 \times 10^3$ s. Therefore, the timescales satisfy the condition $t_{CDI} \ll t_{esc}$, implying that the curvature drift instability is efficient enough to rectify the magnetic field lines (curvature tends to zero), leading to a negligible role of the curvature emission process in the observed VHE domain. It is worth noting that the wave stays in the active zone longer than the plasma which caused this wave. On the other hand, the CDI is a continuous process in spite of the fact that the mode escapes the magnetosphere, because a new portion of plasma excites the CDI again and the process is continuously maintained.

By analyzing the inverse Compton scattering, we have found that for Crab pulsar's magnetospheric parameters even very energetic electrons are unable to produce the observed photon energies.

The emission model proposed in previous works Machabeli & Osmanov (2009, 2010) and developed in the present paper ensures the simultaneous generation of the low and high frequency waves in the same area of the magnetosphere. The distribution function of relativistic particles is one dimensional at the pulsar surface, but plasma with an anisotropic distribution function is unstable which inevitably leads to the wave excitation. The main mechanism of the wave generation in plasmas of the pulsar magnetosphere is the cyclotron

instability, which develops near the light cylinder. During the quasi-linear stage of the instability a diffusion of particles arises along and across the magnetic field lines. Therefore, plasma particles acquire transverse momenta and, as a result, the synchrotron mechanism is switched on. If the resonant particles are the primary beam electrons with $\gamma_b \simeq 6 \cdot 10^8$ their synchrotron emission comes in the high energy domain ($\sim 10\text{GeV}$). The frequency of the original waves, excited during the cyclotron resonance can be estimated from Eq. (3) as follows $\omega_0 \approx \omega_B / \delta \gamma_b$. Estimations show that for the beam electrons with the Lorentz-factor from the interval $\gamma_b \sim 10^{6-8}$, the radio waves are excited. Consequently, we explain the coincidence of radio and γ -ray signals.

We provide the theoretical confirmation of the measured power-law spectrum ($F_\epsilon \propto \epsilon^{-\beta}$ with $\beta = 2$) in the energy domain $\epsilon = 0.01\text{GeV}$ to 25GeV . Differently from the standard theory of the synchrotron emission (Ginzburg 1981), which only explains the spectral index, $\beta < 1$, our approach gives the possibility to obtain the values of index that are much higher than one. The main reason for this is that we take into account the mechanism of creation of the pitch angles, and obtain a certain distribution function of the emitting particles from their perpendicular momenta (see Eq. (10)), which restricts the possible values of the pitch angles. The emission comes from a region of the pulsar magnetosphere where the magnetic field lines are practically straight and parallel to each other. But in the standard theory of the synchrotron emission (Ginzburg 1981), it is supposed that the observed radiation is collected from a large spacial region in various parts of which, the magnetic field is oriented randomly. Thus, it is supposed that along the line of sight the magnetic field directions are chaotic and when finding emission flux, Eq. (14) is averaged over all directions of the magnetic field (which means integration over ψ varying from 0 to π). The measured decrease of the flux at $\epsilon = 25\text{GeV}$ is also explained. Our theoretical spectrum $F_\epsilon \propto \epsilon^{-2} \exp[-(\epsilon/23)^{1.6}]$ yields the exponential cutoff, with the cutoff energy 23GeV .

5. Summary

1. Constructing a self-consistent theory, we interpret the observations of the MAGIC Cherenkov Telescope of the pulsed emission, $(0.01 - 25)\text{GeV}$, from the Crab pulsar.
2. It is emphasized that due to very small cooling timescales, particles rapidly transit to the ground Landau state vanishing the subsequent synchrotron radiation. The situation changes thanks to the cyclotron instability, which efficiently develops on the light cylinder scales and creates non-vanishing pitch angles, leading to the efficient synchrotron process.
3. The observational fact of the coincidence of signals in low (radio) and VHE domains is explained. The original waves excited during the cyclotron instability come in the radio band. It is shown that the resonant electrons interact with the aforementioned waves via QLD, acquire pitch angles and start to radiate in the synchrotron regime.
4. Considering a new approach of the synchrotron theory based on our emission model, we have found the spectral index, β , of VHE emission to be equal to 2 and the exponential cutoff, with the cutoff energy - 23GeV , being in a good agreement with the observational data (Aliu et al. 2008).

In the present paper, we have shown the spacial coincidence of the region of generation of radio and VHE emission of the Crab pulsar. Although, the aforementioned coincidence of signals is detected for broad frequency ranges (from radio up to hard γ -rays). Thus, we suppose that the generation of the multiwavelength radiation of the Crab pulsar takes place in one location of the pulsar magnetosphere. In particular, if the cyclotron resonance occurs for the tail electrons, the above described processes might cause the simultaneous generation of waves in a different energy domains, which is a topic of our future work.

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